Lattice-controlled modulation instability in photorefractive feedback systems

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We study the modulation instability in a two-dimensional nonlinear single feedback system with a photonic lattice and reveal a sharp transition in the instability regimes as the lattice strength is increased. For a shallow lattice, the instability modes are enhanced parallel to the lattice wave vector, while in stronger lattices, these modes are suppressed. © 2010 Optical Society of America

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Periodic photonic structures enable novel methods for manipulation of light emission and radiation. The interplay between periodicity, nonlinearity, and optical gain gives rise to a range of new phenomena, including pattern formation [1–6]. In the field of nonlinear periodic dissipative systems, the control of instability modes forms one of the most challenging problems [7]. Whereas theoretical studies of modulation instability (MI) in feedback systems have been performed for purely one-dimensional (1D) periodic systems [2,3,8], all existing experiments are intrinsically two-dimensional (2D) [5,6]. The 2D instability modes exist due to nonlinear coupling of wave vectors in the entire 2D plane, and therefore the 1D picture cannot be applied to the analysis of such 2D systems.

In this Letter we present a theoretical study of 2D MI in a single feedback system with photorefractive (PR) two-wave mixing gain and an embedded 1D photonic lattice [Fig. 1(a)], revealing a sharp transition between two fundamentally different instability regimes. (i) In a shallow lattice, the instability modes are enhanced along the lattice, and suppressed transversely to the lattice; whereas (ii) for stronger lattices, the instability is suppressed along the lattice, and allowed in the transverse direction. Our results explain the origin of the 2D pattern transformations observed in recent experiments [5,6] and suggest further possibilities for nonlinear pattern control.

We extend the standard model for the evolution of complex envelopes of the forward (F) and backward (B) propagating beams inside a PR crystal [9,10], taking into account the effect of the photonic lattice:

\[ \partial_t F - i D \nabla^2_F F - i V(x,y) F = -Q B, \]

\[ \partial_t B + i D \nabla^2_B B + i V(x,y) B = -Q^* F. \]  

(1)

Here \( \partial_t \) represents a derivative in \( z, D = z_\lambda \partial_\lambda / (4z_\lambda n_0 z_\lambda^2) \) is the diffraction coefficient; \( V = 2\pi z_\lambda \Delta n_L, \Delta n_L(x,y) \) is the refractive-index modulation induced by the photonic lattice; \( n_0 \) is the homogeneous refractive index; \( \lambda_\lambda \) is the laser wavelength in vacuum; \( \nabla^2_F \) is the Laplacian operator in \( (x, y) \) space; and \( x, y \) and \( z_\lambda \) are the scaling distances in transverse and propagation directions, respectively.

The function \( Q \) defines the strength of the PR grating, which evolution in time is modeled following [9] with the addition of the photonic lattice intensity \( I_L \):

\[ \tau_{PR} \partial_t Q + Q = \Gamma FB^* / (I + I_d + I_L). \]  

(2)

Here \( I = |F|^2 + |B|^2; I_d \) is the dark intensity (determined by the PR crystal properties); \( \tau_{PR} \) is the PR relaxation time; \( \Gamma \) is the PR coupling coefficient; and \( \partial_t \) is a derivative in \( t \). We employ the weak modulation depth approximation [9,10], assuming that \( \Gamma \) is intensity and position independent. We consider the case of a 1D photonic lattice with a modulation period \( d \) in the \( x \) direction, i.e., \( I_L(x,y,z) = I_L(x + d,0,0) \), normally incident plane-wave pump \( F(x,y,0) = F_{in} \), and a fully reflecting mirror.

Fig. 1. (Color online) (a) Schematic illustration of a PR feedback system with a photonic lattice. (b)–(d) Forward-wave intensity profiles for the stationary solutions. (e)–(g) Corresponding domains of instability, shown with black shading in the plane of transverse wave vectors \( (k_x, k_y) \). The vertical dashed lines represent the edge of the lattice Brillouin zone. Parameters are (b), (e) \( \Delta n_p = 0, \Gamma = 3.83 \); (c), (f) \( \Delta n_p = 10^{-4}, \Gamma = 3.75 \); and (d), (g) \( \Delta n_p = 4 \times 10^{-5}, \Gamma = 3.95 \). © 2010 Optical Society of America
aligned with the back crystal facet: \( B(x, y, L) = F(x, y, L) \), where \( L \) is the crystal length.

First, we look for stationary solutions of Eqs. (1) and (2). Because of the periodicity of the lattice and the symmetry of the boundary conditions, we seek stationary solutions that have periodic profiles in \( x \) and are homogeneous in the \( y \) direction, \( \{F, B, Q\}(x, y, z) = \{F, B, Q\}(x + d, 0, z) \). We decompose the profiles in Fourier series, \( \{F, B, Q\}(x, y, z) = \sum_m \{F, B, Q\}_m(z) \times \exp(iK_m x) \), where \( K_m = 2\pi m/d \). After substituting these expressions into Eqs. (1) and (2), we obtain an infinite set of coupled equations for the Fourier amplitudes. These equations are complemented with the boundary conditions, expressed as \( F_0(0) = F_{\text{inv}}, F_m(0) = 0, \) and \( B_m(L) = F_m(L) \). For the numerical analysis, the equations are truncated to \( M \) primary harmonics, \(-M/2+1 \leq m \leq M/2\). The equations are solved iteratively by repeating the following steps: (i) determining \( F_m \) and \( B_m \) for fixed \( Q_m \); and (ii) updating \( Q_m \) based on the calculated \( F_m, B_m \) using a relaxation algorithm [10].

For our numerical simulations, we use the parameter values matching the experimental conditions of [5]: \( n_0 = 2.2, \quad L = 10, \quad d = 25, \quad \lambda_0 = 532 \text{ nm}, \quad z_0 = 1 \text{ mm}, \quad x_0 = 1 \mu \text{m} \). The photonic lattice is induced by two plane waves of intensity \( I_p \), Fig. 1(a); hence the lattice intensity profile is \( I_L = 4I_p \cos^2(\pi x/d) \). We can then approximate the induced refractive-index modulation as \( \Delta n_p = \Delta n_p \cos^2(\pi x/d) \) [11], where \( \Delta n_p \) is the refractive-index contrast. The refractive-index modulation along \( z \) plays the most important role in the instability dynamics and therefore, for simplicity, we ignore the changes of the grating due to the lattice, as well as the effect of dark intensity: \( I_p = I_0 = 0 \). In Figs. 1(b)–1(d), we show the obtained characteristic stationary intensity profiles of the forward wave for three lattice modulations.

The stationary solutions can exhibit MI, which leads to the formation of 2D patterns [5,6,9,10]. The instability threshold corresponds to the appearance of perturbations that exactly satisfy the governing equations [9] (i.e., such perturbations have zero growth rate). We consider small changes to the exact stationary solutions, \( F \rightarrow F + f \) and \( B \rightarrow B + b \), and linearize the stationary equations with respect to the small perturbations \( f, b \). The general solution of the linearized equations is decomposed into Floquet-Bloch modes, \( \{f, b\}(x, y, z) = \sum_m \{f, b\}_m(z) \exp(iK_m x) \exp(ik_x x + ik_y y) + \sum_m \{f, b\}_m^*(z) \exp(-iK_m x) \exp(-ik_x x - ik_y y) \). Here \( (k_x, k_y) \) is the perturbation wave vector. Using the boundary conditions at the mirror \( b_{m\pm}(L) = f_{m\pm}(L) \), we express the perturbation amplitudes at the input facet of the crystal through the amplitudes at the mirror using transfer matrices whose elements can be calculated numerically, specifically \( T_{f0}(k_x, k_y) \cdot \{f_{m\pm}(L)\} = \{f_{m\pm}(0)\} \). The boundary conditions at the input facet are \( f_{m\pm}(z = 0) = 0 \), and therefore a nontrivial solution exists only if \( \rho(k_x, k_y) = \det T_{f0}(k_x, k_y) = 0 \). This equation defines the MI threshold for perturbations with specific wave vectors. We have checked that for real \( \Gamma \), \( \im\rho(k_x, k_y) = 0 \). Therefore we plot the sign of \( \rho \) in the plane of perturbation wave vectors \((k_x, k_y)\), such that the boundaries between different regions (where \( \rho \) passes through zero) correspond to the MI threshold. Results for different lattice modulation parameters are presented in Figs. 1(e)–1(g), where in each case we selected a value of \( \Gamma \) slightly above the MI threshold.

When the lattice is absent, the instability region in \( k \)-space has a ring shape [Fig. 1(e)], reflecting the transverse rotational symmetry. The instability appears for a PR coupling constant \( \Gamma \approx 3.5 \), in agreement with previous studies [9,10]. However, when the amplitudes of the modes corresponding to unstable wave vectors grow sufficiently high, the modes start to interact with each other and transverse patterns, such as hexagons, are formed. This interaction cannot be described by the linear stability analysis considered above, and we therefore perform direct numerical simulations of Eqs. (1) and (2), using a beam propagation method combined with an iterative procedure, to update the function \( Q \) [10]. Figures 2(a), 2(d), and 2(g) respectively show the initial, intermediate, and final stages of pattern formation. The resulting hexagonal pattern does not have any preferred orientation in the \((k_x, k_y)\) plane, in agreement with previous theoretical studies [10]. The particular orientation shown in Figs. 2(d) and 2(g) is obtained by introducing a small perturbation in the reflectivity of the PR grating.
formed by the forward and backward propagating modes. For comparison, in Fig. 2(j), we present the experimentally recorded k-space image extracted from the set of measurements performed in our earlier experiments [5].

The presence of the photonic lattice fundamentally alters the instability properties of the system. Figure 1(f) presents the instability domains for a lattice with a shallow refractive-index contrast $\Delta n_p = 10^{-5}$. The length of the lattice wave vector is twice the length of the wave vector associated to the the hexagonal pattern in Fig. 2(d), as under such conditions, the lattice facilitates strong coupling between the MI modes. Because of the lattice periodicity, the instability regions are also periodic in $k_x$. First, the lattice decreases the instability threshold to $\Gamma \approx 3.75$ in Fig. 1(f), compared to $\Gamma \approx 3.8$ without the lattice. Most importantly, the lattice suppresses some regions of instability on the ring in a direction transverse to the lattice orientation, whereas the instability modes along the lattice are still allowed. As a result, the rotational symmetry is removed and only a single pattern orientation is allowed. This is confirmed by direct numerical simulations, shown in Figs. 2(b), 2(e), and 2(h). The far field of the pattern beam diffraction on the lattice gives rise to the two outer spots (marked with circles) appearing along the lattice direction and located exactly at twice the hexagonal pattern $k$-vector. The hexagonal pattern in Figs. 2(e) and 2(h) is rotated by 30° with respect to the hexagons in Figs. 2(d) and 2(g) due to the presence of instability modes along the lattice wave vector at $k_y = 0$ and the inhibition of the modes in the transverse direction [see Fig. 1(D)]. This conclusion agrees well with the experimental patterns shown in Fig. 2(k). We note that while this rotation of the hexagonal pattern remained unnoticed in [5], it becomes clearly visible under the direct comparison in Fig. 2.

For a stronger lattice with refractive-index contrast $\Delta n_p = 4 \times 10^{-5}$, the instability regions change completely in comparison to the shallow lattice; see Fig. 1(g). Similar to the 1D case, the modes along the lattice wave vector are now suppressed, while allowed in the transverse direction. The shape of the instability region resembles a photonic band diagram and the mode suppression is associated with the photonic bandgap effect. This leads to a pattern formation shown in Figs. 2(c), 2(f), and 2(i). Again, this result is in good agreement with experiments [Fig. 2(i)]. We note that whereas in experiments [5], the pattern formation was controlled by varying the beam intensity, which corresponds to the regime of high modulation depth for the induced PR grating [12], the good agreement with our theoretical results obtained in the weak modulation depth approximation indicates that the lattice-controlled pattern transformations are structurally stable.

In summary, we have performed a detailed theoretical analysis of modulation instability and pattern formation in a 2D PR feedback system in the presence of a photonic lattice. We have shown that the lattice depth plays a non-trivial role in the pattern development: for a shallow lattice, the instability modes along the lattice direction can be enhanced, whereas they are suppressed in a stronger lattice, providing a theoretical description of recent experimental observations [5]. We stress that the lattice depth controlled transition of MI has no known analogs in feedback systems with phase modulation [1] or Fourier filtering [13] elements placed outside the PR crystal. Our results provide theoretical description of recent experimental observations [5] and suggest new opportunities for control of nonlinear modes in periodic dissipative systems. We believe that similar effects can be observed in other nonlinear systems, including photonic crystal lasers.

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References