Profile of photorefractive one-dimensional bright spatial solitons

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Analytic solutions for the profile of one-dimensional bright photorefractive steady-state spatial solitons generated as a result of screening of drift or photogalvanic currents are found by means of a power series development. The solutions are valid over all ranges of intensities. We also give simple analytic expressions that describe the soliton full width at half-maximum as a function of all important experimental parameters.

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If an optical beam of a proper wavelength traverses an electro-optic photorefractive crystal charge carriers are photoexcited and redistributed in the vicinity of the beam. In the case of a local nonlinearity the related modifications of the refractive index induce either a self-focusing or a self-defocusing of the beam itself. With proper choice of the experimental parameters the input beam profile can converge asymptotically to a spatial soliton state for which the photorefractive nonlinearity exactly compensates diffraction and the beam propagates unchanged. The big interest in photorefractive spatial solitons stems from the low optical power (microwatts) required. The most common and practical form of photorefractive solitons is the so-called dark intensity level. The dark intensity

\[ I_d \]

can be related also to an auxiliary illumination of a different wavelength than the beam. It is normalized such that for

\[ I(x,z) = |A(x,z)|^2 \]

the parameter

\[ I_{max}/I_D \ll 1 \]

is the expression for the FWHM of the soliton as a function of the intensity-to-dark-intensity ratio, the applied field, and (or) the photogalvanic field only. The present approach gives a complete description of 1D bright solitons without relying on numerical calculations.

We assume an optical beam propagating in the z direction and having infinite extension in the y direction. Such a wave is of the form

\[ E(x,z) = \hat{e}A(x,z)\exp(ikz), \]

where \( \hat{e} \) is a unit vector, \( A(x,z) \) is the complex scalar wave amplitude, and \( k = k_0n = 2\pi n/\lambda \) is the wave vector of the optical wave with vacuum wavelength \( \lambda \) propagating in a medium with refractive index \( n \). In the paraxial approximation and under the influence of a photorefractive nonlinearity the beam propagation is governed by

\[ i\frac{\partial}{\partial z} A(x,z) + \frac{1}{2k_0n} \frac{\partial^2}{\partial x^2} A(x,z) + \frac{k_0}{2} n^3 r_{eff} E_{sc}(x,z) A(x,z) = 0, \]

where \( E_{sc} \) is the photorefractive space-charge field and \( r_{eff} \) is an effective electro-optic coefficient. In its simplest form

\[ E_{sc}(x,z) = E_{ph}(x) = (E_0 + E_{ph}) \frac{I_D}{I(x,z) + I_D}, \]

where \( I(x,z) \propto |A(x,z)|^2 \) is the light intensity in the beam, \( E_0 \) is the externally applied electric field, \( E_{ph} \) is the field that would generate a photocurrent equal to the photogalvanic current at a given light-intensity level. The dark intensity \( I_D \) can be related also to an auxiliary illumination of a different wavelength than the beam. It is normalized such that for

\[ I(x,z) = I_D \]

an equal number of carriers are generated by the beam \( |I(x,z)| \) and by the auxiliary light and (or) thermal excitation \( (I_p) \). Equation (2) is obtained by starting from the set of initial equations that describe the photorefractive effect, under the assumption that the minimum beam size is large compared with the Debye screening length \( L_{Debye} \), which is typically 0.5 \( \mu \)m or less in usual photorefractive crystals. The spatially constant term \( E_{ph} \) on the left-hand side of Eq. (2) produces only a homogeneous offset of the refractive index and can be neglected below.

Equation (1) can have a variety of solutions: Here we look for a soliton solution of the form

\[ A(x,z) = \sqrt{I_D} u(x)\exp(i\Gamma z), \]

where \( \Gamma = \sqrt{\frac{\alpha I_D}{2\pi n^2 k_0}} \) is the propagation constant of the soliton, and \( u(x) \) is a real function that satisfies

\[ \frac{d^2 u}{dx^2} + \alpha u(x) = 0, \]

where \( \alpha = \frac{2\pi n^2 k_0}{\lambda^2} \) is the diffraction constant. This equation is solved by numerical calculations to obtain the soliton shape as a function of the beam intensity.
where \( u(x) \) is a normalized real amplitude independent of the propagation distance \( z \) and \( \Gamma \) gives the mismatch between the propagation wave vectors of the soliton and a plane wave. In the absence of excessive transversal instabilities\(^\text{12} \) a beam will eventually be attracted to the form of Eq. (3) after all spatial transients, which may exceed in many cases the typical crystal lengths, have damped out.\(^\text{13} \) Inserting Eq. (3) into the nonlinear paraxial wave Eq. (1) with \( E_{\text{sc}}(x,z) \) according to Eq. (2) and multiplying by \( 2k_0n \), one gets

\[
u'' - \left( \alpha + \frac{\beta}{u^2} \right) u = 0, \quad (4)
\]

where \( u'' \) is the second derivative with respect to \( x \) and

\[
\alpha = 2k_0n \Gamma, \quad \beta = k_0^2 n^4 \epsilon_{\text{eff}} (E_0 + E_{\text{ph}}). \quad (5)
\]

Equation (4) describes the profile \( u(x) \) of a photorefractive spatial soliton and has the form of the nonlinear Schrödinger equation with a saturable nonlinearity.\(^\text{14} \) Bright solitons are characterized by the boundary conditions

\[
u(x = 0) = u_0, \quad u(x \to \pm \infty) = 0,
\]

\[
u'(x = 0) = u'(x \to \pm \infty) = 0, \quad (6)
\]

and solutions of this kind exist for \( \beta > 0 \). Equation (4) can be integrated once, and the boundary conditions give us the relationship

\[
\alpha = - \frac{\beta}{u_0^2} \ln(1 + u_0^2). \quad (7)
\]

\[
a_6 = \frac{18a_2a_4 + 10a_0a_2^3 - 10a_0^2a_2a_4 - \alpha(2a_0a_4 + a_2^2) - (\alpha + \beta)(6a_0^2a_2^2 + 4a_0^3a_4)}{30a_0(1 + a_0^2)}, \quad (15)
\]

where the value of \( \alpha \) is always obtained from \( \beta \) by use of Eq. (7).

To show the quick convergence of our development, we plot in Fig. 1 the bright soliton beam profile calculated with Eq. (9). Using the three terms up to \( a_4 \) or the four terms up to \( a_6 \). The curves are compared with the exact profile (circles). For \( u_0 = 0.01 \) and \( u_0 = I_{\text{max}}/I_D = 1 \) the agreement is nearly perfect. For the largest intensity ratio \( u_0 = 100 \) higher-order terms would be necessary to describe correctly the borders of the soliton beam, where the intensity is less than 5% of the maximum. However, even in this case the width at half-maximum is described with an accuracy of 3.5% with three terms and 1.5% with four terms.

One can obtain a simple expression for the FWHM of the soliton beam as a function of experimental parameters by considering only the terms up to \( a_4 \). The HWHM is given by the roots of the biquadratic equation \( \alpha_0 + a_2x^2 + a_4x^4 = u_0\sqrt{2} \), which leads to

\[
\text{FWHM} = \left( \frac{2}{a_4} \right) \left[ 1 + 4(\sqrt{2} - 1) \frac{\alpha_0a_4}{a_2^2} \right]^{1/2} - 1 \right)^{1/2}. \quad (16)
\]

Figure 2 shows the FWHM versus the parameter \( u_0 = (I_{\text{max}}/I_D)^{1/2} \) for three values of the coupling constant.
Fig. 1. Soliton profile for three values of $u_0 = (I_{\text{max}}/I_D)^{1/2}$. Circles, exact solution as obtained from Eq. (8) for $u_0 = 0.01$ and by numerical integration of Eq. (4) otherwise; solid curves, power series solution using the terms in $a_0$, $a_2$, and $a_4$ in Eq. (9); dashed curves, power series solution using in addition the term in $a_6$. Note the different $x$-axes applying to the different profiles.

Fig. 2. Soliton FWHM as a function of $u_0$, as calculated from Eq. (16).

Fig. 3. Required nonlinearity factor $\beta$ to generate a soliton of a given width as a function of $u_0$. The right-hand scale gives the field required in KNbO$_3$ crystals in a geometry that uses the electro-optic coefficient $r_{33,\text{eff}} = 54 \text{ pm/V}$ at $\lambda = 633$ nm.

The minimum width of 1D bright solitons is found for $I_{\text{max}}/I_D = 2\text{--}3$ and is independent of $\beta$. Using Eqs. (12)\textendash(14) and Eq. (16) one can also calculate the required electric field needed to generate a soliton with a given width and intensity ratio $I_{\text{max}}/I_D$. This is shown in Fig. 3, in which the right-hand scale gives the field required in KNbO$_3$ in a geometry that uses only the electro-optic coefficient $r_{33} = 54 \text{ pm/V}$ at $\lambda = 633$ nm.

might be observed for the same sets of parameters. Such a behavior is the signature of nonstationary self-focusing only.

The method presented here can also be used to describe dark solitons as well as two-dimensional spatial solitons.

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References